1. A company has a large number of regular users logging onto its website. On average 4 users every hour fail to connect to the company's website at their first attempt.

(a) Explain why the Poisson distribution may be a suitable model in this case.

(1)

Find the probability that, in a randomly chosen **2 hour** period,

(b) (i) all users connect at their first attempt,

(ii) at least 4 users fail to connect at their first attempt.

(5)

The company suffered from a virus infecting its computer system. During this infection it was found that the number of users failing to connect at their first attempt, over a 12 hour period, was 60.

(c) Using a suitable approximation, test whether or not the mean number of users per hour who failed to connect at their first attempt had increased. Use a 5% level of significance and state your hypotheses clearly.

(9)
(Total 15 marks)

2. A report states that employees spend, on average, 80 minutes every working day on personal use of the Internet. A company takes a random sample of 100 employees and finds their mean personal Internet use is 83 minutes with a standard deviation of 15 minutes. The company's managing director claims that his employees spend more time on average on personal use of the Internet than the report states.

Test, at the 5% level of significance, the managing director's claim. State your hypotheses clearly.

(Total 7 marks)

1. (a) Connecting occurs at random/independently, singly or at a constant rate B1 1

Note

B1 Any one of randomly/independently/singly/constant rate. Must have context of connection/logging on/fail

(b) Po (8) B1

Note

B1 Writing or using Po(8) in (i) or (ii)

(i) $P(X = 0) = 0.0003$ A1

Note

for writing or finding $P(X = 0)$
A1 awrt 0.0003

(ii) $P(X \geq 4) = 1 - P(X \leq 3)$
$= 1 - 0.0424$
$= 0.9576$ A1 5

Note

for writing or finding $1 - P(X \leq 3)$
A1 awrt 0.958

(c) $H_0 : \lambda = 4$ (48) $H_1 : \lambda > 4$ (48) B1

N(48, 48) A1

Method 1

$P(X \geq 59.5) = P\left(Z \geq \frac{59.5 - 48}{\sqrt{48}}\right)$

$= P(Z \geq 1.66)$
$= 1 - 0.9515$
$= 0.0485$

Method 2

$\frac{x - 0.5 - 48}{\sqrt{48}} = 1.6449$ A1

$x = 59.9$ A1

$0.0485 < 0.05$
Reject H_0. Significant. 60 lies in the Critical region
The number of failed connections at the first attempt has increased. A1 ft 9

Note

B1 both hypotheses correct. Must use λ or μ
identifying normal
A1 using or seeing mean and variance of 48
These first two marks may be given if the following are seen in the standardisation formula : 48 and $\sqrt{48}$ or awrt 6.93
for attempting a continuity correction (Method 1: 60 ± 0.5 / Method 2: $x \pm 0.5$)
for standardising using their mean and their standard deviation and using either Method 1 [59.5, 60 or 60.5. accept ± z.] Method 2 [($x \pm 0.5$) and equal to a ± z value)
A1 correct z value awrt ±1.66 or ± $\frac{59.5-48}{\sqrt{48}}$, or $\frac{x-0.5-48}{\sqrt{48}}=1.6449$
A1 awrt 3 sig fig in range 0.0484 – 0.0485, awrt 59.9
for "reject H_0" or "significant" maybe implied by "correct contextual comment"
If one tail hypotheses given follow through "their prob" and 0.05, $p < 0.5$
If two tail hypotheses given follow through "their prob" with 0.025, $p < 0.5$
If one tail hypotheses given follow through "their prob" and 0.95, $p > 0.5$
If two tail hypotheses given follow through "their prob" with 0.975, $p > 0.5$
If no H_1 given they get M0
A1 ft correct contextual statement followed through from their prob and H_1. need the words number of failed connections/log ons has increased o.e.
Allow "there are more failed connections"
NB A correct contextual statement **alone** followed through from their prob and H_1 gets A1

[15]

2. H_0: $\mu = 80$, H_1: $\mu > 80$ B1, B1

$$z = \frac{83 - 80}{\frac{15}{\sqrt{100}}} = 2$$ A1

$2 > 1.6449$ (accept 1.645 or better) B1

Reject H_0 or significant result or in the critical region

Managing director's claim is supported. A1 7

Note

1st B1 for H_0. They must use μ not x, p, λ or $\bar{x}$ etc

2nd B1 for H_1 (must be > 80). Same rules about μ.

1st for attempt at standardising using 83, 80 and $\frac{15}{\sqrt{100}}$.
Can accept ±. May be implied by $z = \pm 2$

1st A1 for + 2 only

3rd B1 for ± 1.6449 seen (or probability of 0.0228 or better)

2nd for a correct statement about "significance" or rejecting H_0 (or H_1) based on their z value and their 1.6449 (provided it is a recognizable critical value from normal tables) or their probability (< 0.5) and significance level of 0.05.
Condone their probability > 0.5 compared with 0.95 for the 2nd

2nd A1 for a correct contextualised comment. Must mention "director" and "claim" or "time" and "use of Internet". No follow through.

2nd M1A1

If no comparison or statement is made but a correct contextualised comment is given the can be implied.
If a comparison is made it must be compatible with statement otherwise M0
e.g. comparing 0.0228 with 1.6449 is M0 or comparing probability 0.9772 with 0.05 is M0
comparing –2 with – 1.6449 is OK provided a correct statement accompanies it
condone –2 > –1.6449 provided their statement correctly rejects H_0.

Critical Region

They may find a critical region for $\overline{X}$: $\overline{X} > 80 + \frac{15}{\sqrt{100}}$

$\times 1.6449$ = awrt 82.5

1st for $80 + \frac{15}{\sqrt{100}} \times$ (z value)

3rd B1 for 1.645 or better

1st A1 for awrt 82.5

The rest of the marks are as per the scheme.

[7]

1. The majority of candidates were familiar with the technical terms in part (a), but failed to establish any context.

Part (b) was a useful source of marks for a large proportion of the candidates. The only problems were occasional errors in detail. In part (i) a few did not spot the change in time scale and used Po(4) rather than Po(8). Some were confused by the wording and calculated P($X = 8$) rather than P($X = 0$). The main source of error for (ii) was to find $1 - \text{P}(X \leq 4)$ instead of $1 - \text{P}(X \leq 3)$.

In part (c) the Normal distribution was a well-rehearsed routine for many candidates with many candidates concluding the question with a clear statement in context.
The main errors were

- Some other letter (or none) in place of λ or μ
- Incorrect Normal distribution: e.g. N(60, 60)
- Omission of (or an incorrect) continuity correction
- Using 48 instead of 60
- Calculation errors

A minority of candidates who used the wrong distribution (usually Poisson) were still able to earn the final two marks in the many cases when clear working was shown. This question was generally well done with many candidates scoring full marks.

2. This was a straightforward starter to the paper and many fully correct solutions were seen. The usual problems with the hypotheses were present (using $\bar{x}$ not μ, thinking $\mu = 80$ not 83) but the calculation was often correct and a correct statement and conclusion in context usually followed. A small minority compared a *z* value with a probability but this error was rare at this level.